P-wave heavy-light mesons using NRQCD and D234

Randy Lewis[a] and R. M. Woloshyn[b]

[a]Department of Physics, University of Regina, Regina, SK, Canada S4S 0A2

[b]TRIUMF, 4004 Wesbrook Mall, Vancouver, BC, Canada V6T 2A3

The masses of S- and P-wave heavy-light mesons are computed in quenched QCD using a classically and tadpole-improved action on anisotropic lattices. Of particular interest are the splittings among P-wave states, which have not yet been resolved experimentally; even the ordering of these states continues to be discussed in the literature. The present work leads to upper bounds for these splittings, and is suggestive, but not conclusive, about the ordering.

1. INTRODUCTION

The masses of S-wave heavy-light mesons have been a valuable testing ground for lattice NRQCD studies. The calculations are precise enough to test convergence of the NRQCD expansion[1], and the effects of quenching[2]. Also, experimental measurements are precise enough to constrain the values of coefficients in the NRQCD action[2].

For P-waves, the situation is quite different. Experiments have not yet resolved the splittings among P-wave states, and Isgur[3] has recently promoted the suggestion, initiated long ago by Schnitzer[4], that the P-wave states may be inverted relative to the conventional hydrogen-like ordering. This inversion is predicted by using the experimental K^* masses and mixing angle as input, then extrapolating from the constituent strange quark to charm or bottom via a $1/M$-expanded nonrelativistic potential. Verification of this prediction would thus lend support to the treatment of a constituent strange quark as "heavy" within a nonrelativistic quark model.

To date, quenched lattice QCD calculations have not seen an inversion[5–7], although the uncertainties are often large enough to make firm conclusions difficult.

The present work is an exploration of this physics using an improved action on anisotropic lattices. The results obtained below agree qualitatively with previous lattice calculations, but a quantitative comparison reveals differences.

2. ACTION

The lattice action has three terms: gauge action, light quark action and heavy quark action. The entire action is classically and tadpole-improved with the tadpole factors, $U_{0,s}$ and $U_{0,t}$, defined as the mean links in Landau gauge in a spatial and temporal direction, respectively.

The gauge action includes a sum over $1{\times}2$ rectangular plaquettes as well as $1{\times}1$ elementary plaquettes, and therefore the leading classical errors are quartic in lattice spacing. For light quarks, a D234 action[8] is used with parameters set to their tadpole-improved classical values. Its leading errors are cubic in lattice spacing.

The heavy quark action is NRQCD[9], which is discretized to give the following Green's function propagation[1]:

$$G_{\tau+1} = \left(1-\frac{a_tH}{2n}\right)^n \frac{U_4^\dagger}{U_{0,t}} \left(1-\frac{a_tH}{2n}\right)^n G_\tau, \quad (1)$$

with $n=5$ chosen for this work, and the Hamiltonian is truncated at $O(1/M)$,

$$H = \frac{-\Delta^{(2)}}{2M} - \frac{1}{U_{0,s}^4}\frac{g}{2M}\boldsymbol{\sigma}\cdot\tilde{\mathbf{B}} + \frac{{a_s}^2\Delta^{(4)}}{24M}, \quad (2)$$

$$\tilde{B}_i = \frac{1}{2}\epsilon_{ijk}\tilde{F}_{jk}. \quad (3)$$

A tilde denotes removal of the leading discretization errors, leaving the action with quadratic lattice spacing errors. Notice that the coefficients in H are set to their tadpole-improved classical values.

Table 1
Heavy-light meson creation operators.

$^{2S+1}L_J$	$\Omega(\vec{x})$
$^1S_0$	$(0,I)$
$^3S_1$	$(0,\sigma_i)$
$^1P_1$	$(0,\Delta_i)$
$^3P_0$	$(0,\sum_i \Delta_i\sigma_i)$
$^3P_1$	$(0,\Delta_i\sigma_j - \Delta_j\sigma_i)$
$^3P_2$	$(0,\Delta_i\sigma_i - \Delta_j\sigma_j)$ or
	$(0,\Delta_i\sigma_j + \Delta_j\sigma_i)$, $i \neq j$

3. RESULTS

All data presented here come from 2000 gauge field configurations on $10^3\times30$ lattices at $\beta = 2.1$ with a bare aspect ratio of $a_s/a_t = 2$, which corresponds to $a_t = 0.10$ fm. The light quark mass is fixed by $\kappa = 0.24$, which gives $m_\pi/m_\rho = 0.52 \pm 0.01$, ie. $m_q \sim m_s/2$. Fixed time boundaries are used for the light quark propagators so they fit naturally into a meson with an NRQCD heavy quark propagator.

Four heavy quark masses have been studied: $a_sM = 1, 3, 5, \infty$. Calculation of the kinetic mass of the $^1S_0$ heavy-light meson leads to the following charm and bottom quark masses: $a_sM_c \sim 1.2$ and $a_sM_b \sim 5$. More precise determinations of these parameters are not required for the present exploratory study.

Familiar heavy-light meson creation operators are used,

$$\sum_{\vec{x}} Q^\dagger(\vec{x})\Omega(\vec{x})\Gamma(\vec{x})q(\vec{x}), \tag{4}$$

where $\Omega(\vec{x})$ is given in table 1 and the smearing operator is

$$\Gamma(\vec{x}) = [1 + c_s\Delta^{(2)}(\vec{x})]^{n_s}. \tag{5}$$

All plots shown here use $(c_s, n_s) = (0.15, 10)$ at the source (which is fixed to timestep 4) and a local sink.

Fig. 1 shows the $^1S_0$ and $^1P_1$ simulation energies in the bottom region. In NRQCD, only energy differences are physical but this plot is an indication of the data quality. As for all plots in this work, the uncertainties are determined from 5000 bootstrap ensembles. It should be noted that the $^1P_1$-$^1S_0$ splitting is in agreement with previous lattice determinations (eg. [5–7]).

Fig. 2 shows the $^3S_1$-$^1S_0$ mass splitting and the kinetic shift of the $^1S_0$ simulation energy in the bottom region. In physical units, the $^3S_1$-$^1S_0$ splitting is found to be 34±5 MeV, in agreement with the known quenched result (eg. [1,5–7]).

Fig. 3 displays the mass splittings among P-wave states at all available quark masses. The $^3P_1$ is not shown since it is not distinguishable from the $^1P_1$. Near the source, the states are organized according to the familiar hydrogen-like pattern: P_0, P_1, P_2 from lightest to heaviest.

As time increases, the uncertainties grow such that one might be tempted to define a plateau which begins rather close to the source. However, the central values of the mass splittings tend to decrease for increasing time, so a negative mass splitting (i.e. an inverted spectrum) cannot be ruled out conclusively.

Using $a_t = 0.10$ fm, Fig. 3 indicates that the $^3P_2$-$^3P_0$ splitting for bottom mesons is less than 100 MeV, which should be contrasted with Ref. [7], where lattice NRQCD gave 183±34 MeV. While it is true that the two determinations stem from different methods (different lattice actions, anisotropic versus isotropic lattices, different light quark masses, slightly different temporal lattice spacings, . . .), it is disconcerting that the final results are not in better agreement.

In conclusion, this work has led to an upper bound for the $^3P_2$-$^3P_0$ (bottom) splitting which lies below a previous lattice determination. An inverted spectrum, although not suggested by the calculation, cannot yet be ruled out.

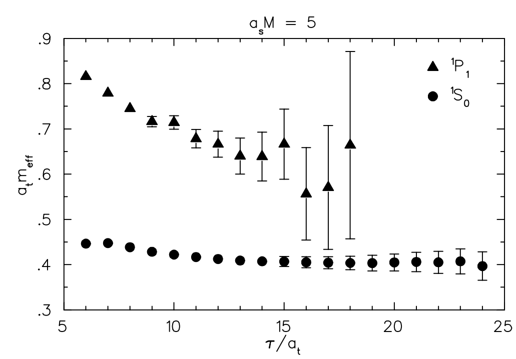

Figure 1. The $^1S_0$ and $^1P_1$ simulation energies in lattice units at $a_sM = 5$.

ACKNOWLEDGEMENTS

R.L. thanks A. Ali Khan, P. Boyle, S. Collins and J. Hein for discussions. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada.

REFERENCES

1. R. Lewis and R. M. Woloshyn, Phys. Rev. D, **58**, 074506 (1998); Nucl. Phys. B (Proc. Suppl.) 73, 333 (1999).
2. S. Collins et. al., preprint hep-lat/9901001 (1999).
3. N. Isgur, Phys. Rev. D, **57**, 4041 (1998).
4. H. J. Schnitzer, Phys. Lett. B, **76**, 461 (1978); Nucl. Phys. B **207**, 131 (1982).
5. C. Michael and J. Peisa, Phys. Rev. D **58**, 034506 (1998).
6. P. Boyle, Nucl. Phys. B (Proc. Suppl.) **63**, 314 (1998).
7. A. Ali Khan et. al., Phys. Lett. B **427**, 132 (1998); A. Ali Khan, Nucl. Phys. B (Proc. Suppl.) **73**, 345, (1999).
8. M. Alford, T. R. Klassen and G. P. Lepage, Nucl. Phys. B **496**, 377 (1997).
9. W. E. Caswell and G. P. Lepage, Phys. Lett. B **167**, 437 (1986); G. P. Lepage and B. A. Thacker, Nucl. Phys. B (Proc. Suppl.) **4**, 199 (1988); B. A. Thacker and G. P. Lepage, Phys. Rev. D **43**, 196 (1991); G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995).

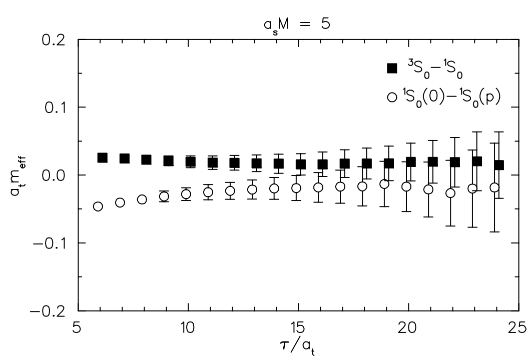

Figure 2. The S-wave mass splittings in lattice units at $a_sM = 5$. The nonzero momentum “p” is $2\pi/(10a_s)$.

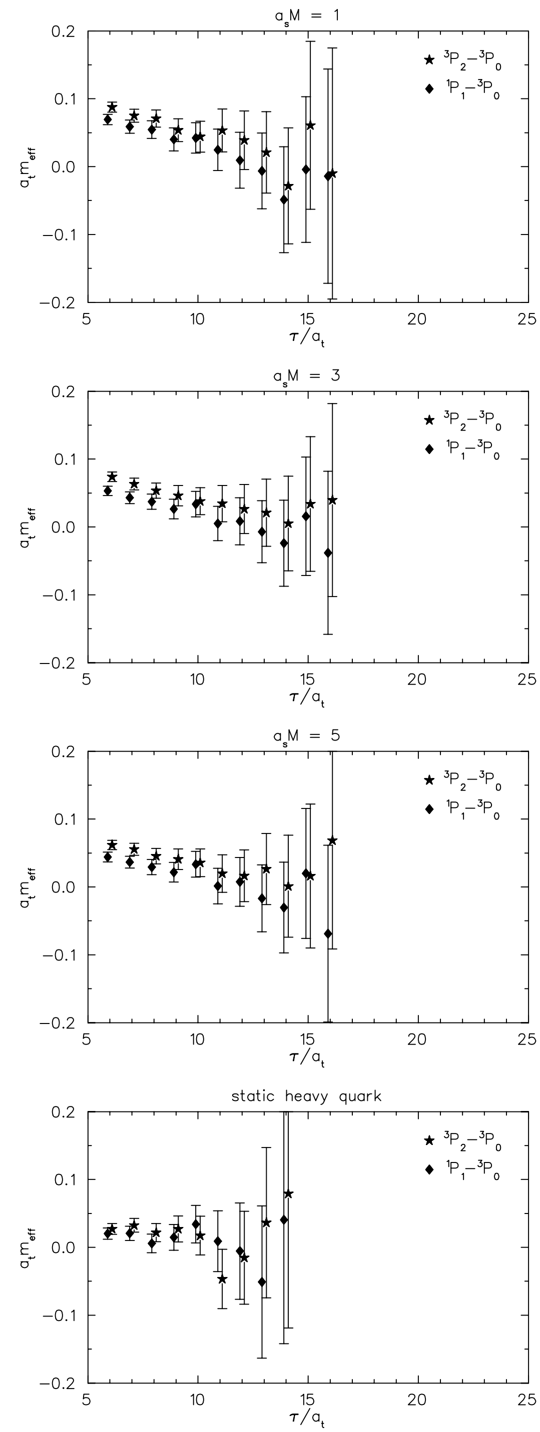

Figure 3. The P-wave mass splittings in lattice units.